IWST 3. The system of linear equations. Method of Gauss.

1. **Tasks: Solve the system of equations with the method of Gauss** :
[3], №№38-50, p.129-130

2. Examples:

We consider a system of three equations with three unknowns

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 2 \\ 3x_1 - 3x_2 + 2x_3 = -6 \\ x_1 + x_2 + x_3 = 2 \end{cases}$$

Solution:

where $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -3 & 2 \\ 1 & 1 & 1 \end{pmatrix}$

$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$; $B = \begin{pmatrix} 2 \\ -6 \\ 2 \end{pmatrix}$; $\Rightarrow A \cdot X = B$ matrix form of the system.

We check the compatibility of the system:
with the method of Gauss

$$A^P = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 2 & -3 & 2 & -6 \\ 1 & 1 & 1 & 2 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -7 & -4 & -10 \\ 0 & -1 & -2 & 0 \end{array}\right) =$$

expanded matrix

$$= \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & -2 & 0 \\ 0 & -7 & -4 & -10 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 10 & -10 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

system is compatible, the determinant of order 3 is different from zero. The system has a unique solution. We can find solutions

$$x_3 = \frac{-1}{1} = -1, \quad x_2 = 0 - 2x_3 = 1, \quad x_1 = 2 - 2x_2 - 3x_3 = 1$$